Multifractal structure and intermittency of laser-generated turbulence in nematic liquid crystals

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We study the chaotic dynamics which occurs in a nematic liquid crystal film during the director reorientation induced by an intense optical field. We show the presence of intermittency in the fluctuations of the transmitted light intensity polarized perpendicularly to the incident beam. The intermittency can safely be described by the multifractal geometry, and, in this framework, we present a simple heuristic model for the phenomenon. [S1063-651X(96)04111-6]

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The interaction of a nematic liquid crystal (NLC) with an intense optical field can exhibit different features depending on the experimental geometries used, i.e., the unperturbed director orientation [1], the light polarization, and the incidence angle. This effect results in several oscillating diffraction rings (which can be observed in the far field pattern of the transmitted beam) in two different geometries: (1) When a circularly or elliptically polarized light beam acts on a homeotropically aligned NLC film at normal incidence [7]; and (2) when a linearly polarized light beam impinges on a homeotropically aligned NLC film at a small incidence angle, and the light polarization is perpendicular to the incidence plane [8,9].

In previous papers [9] we reported the characterization of a very interesting dynamical behavior observed in the latter geometry, with a sample cell consisting in a thermally stabilized ($T=18°$C) film of $E7$. The dynamics of the molecular director have been tracked by instantaneously detecting $I_{\parallel}(t)$ and $I_{\perp}(t)$ that are the intensities of the two components in the center of the outgoing beam polarized parallel and perpendicular to the polarization of the incident beam. The light intensities are detected by two photodiodes, the acquired time series of duration $T=N\Delta\tau$ consists of records of $N=16\,384$ points that are sampled at fixed time interval $\Delta\tau=0.1$ s. A more detailed description of the experimental setup and procedures are reported in Refs. [9]. In these papers we described the dynamical behavior followed by the system which, as the incident power is increased, undergoes an interesting route toward a state where the rings oscillate chaotically.

In recent years another method has been introduced which describes the fine structure of chaotic attractors in the phase space. It consists of determining the multifractal structure of the attractor, if any exists [2]. Multifractals has been introduced to describe processes where anomalous scaling laws are present. Among other main examples we include some chaotic systems [3], intermittent fluid flows [4,5], and astronomical systems [6]. For a review, see Ref. [2]. In the present paper we show that intermittency, leading to anomalous scaling laws, is present in the chaotic state observed in our experiment, and that intermittency can be safely described by the multifractal geometry. Our results, which can be interpreted in the light of a simple toy model, represent a testing bench for a theoretical investigation of light induced reorientation in liquid crystals, thus opening a way to a deeper understanding of the phenomenon.

Let us consider the time series $I(t)$ of duration T in the chaotic regime, and let us divide the whole duration into disjoint subsets $T_i \subset T$ of equal size Δt. Then we can introduce a probability measure through

$$P_{\Delta t}(T_i,I)=\int_{t\in T_i} I(t)dt \Bigg/ \int_0^T I(t)dt, \qquad (1)$$

where the integral in the numerator is extended to the times belonging to the ith subset. Quantity (1) represents the probability of occurrence of a given $I(t)$ in a particular subset T_i at the scale Δt, and is at the heart of our analysis. In Figs. 1 we show the measure densities $S_{\Delta t}(T_i,I) = (T/\Delta t)P_{\Delta t}(T_i,I)$ vs the ith box, both for $I_{\parallel}(t)$ and $I_{\perp}(t)$. It is worth noting some peculiarities of Fig. 1(b) which are not observable in Fig. 1(a), and which show the main difference between $I_{\parallel}(t)$ and $I_{\perp}(t)$. First of all, fluctuations behave similarly on different time scales. A peak in the curve calculated at high Δt values tends to be resolved into two or more peaks in the curve corresponding to smaller Δt values. This evidence of scale invariance is typical of a fractal structure. On the other hand, the bursting behavior exhibited by $S_{\Delta t}(T_i,I_{\perp})$ on all time scales indicates that the system undergoes large fluctuations in a small time interval. In fact, Fig. 1(b) shows the presence in the observed signal of regions characterized by high intensity over small time intervals that are followed by regions of small intensity over large time intervals. These trends indicate that fluctuations are intermittent in time rather than homogeneous. We can also observe that fluctuations are asymmetric with respect to the average intensity, thus reflecting a tail in the distribution, and underlining that higher fluctuations are favored. Finally we stress that fluctuations are larger on smaller scales. The intermittency is less evident in Fig. 1(a), where fluctuations of $S_{\Delta t}(T_i,I_{\parallel})$ are still asymmetric, but the distribution tail is less important, and negative fluctuations are favored.

The presence of a bursting structure in the densities is a phenomenon which is common to intermittent processes, and can safely be described by multifractal geometry. In the multifractal picture one introduces a set of singularities of strength α for the probability measure $P_{\Delta t}(T_i,I)\sim(\Delta t/T)^{\alpha}$ (the symbol $\sim$ means that both sides of the equation have the same scaling law) and then calculates the dimension $f(\alpha)$ of

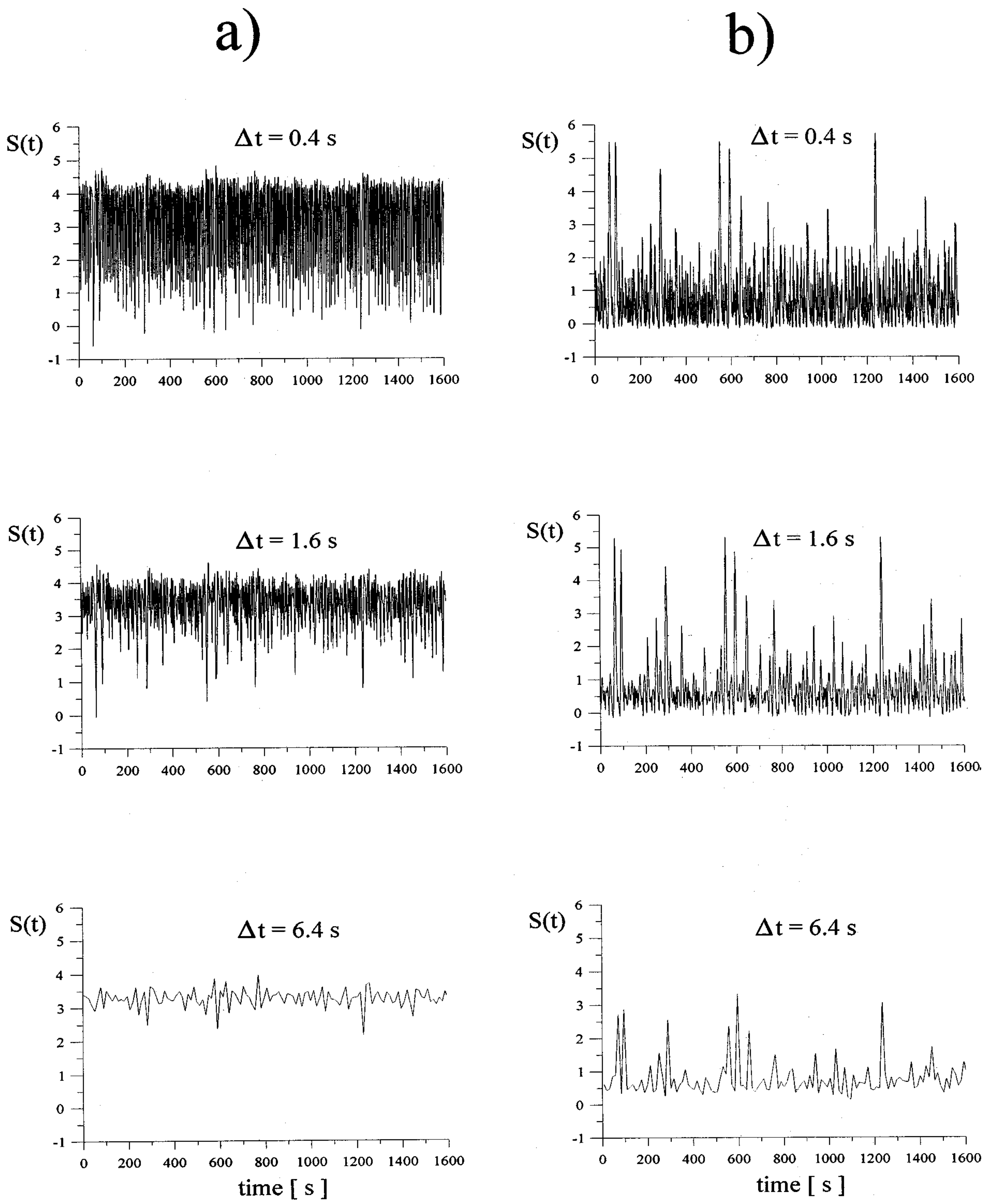

FIG. 1. Time behavior of $S_{\Delta t}(I,t)$ calculated both for $I_{\parallel}$ and $I_{\perp}$ for different time intervals Δt.

the various iso-α sets. To emphasize the strongest spikes, we look for the scaling exponents $\kappa(q)$ of all the moments

$$\sum_i [P_{\Delta t}(T_i,I)]^q \sim (\Delta t/T)^{\kappa(q)} \qquad (2)$$

(the sum is extended to all the subsets T_i). The scaling exponents $\kappa(q)$ are related to the set of generalized dimensions $D_q = \kappa(q)/(q-1)$ [10], and to the multifractal functions (α, f) through the relations $f(\alpha) = q\alpha - (q-1)D_q$ and $\alpha = d/dq[(q-1)D_q]$ [2].

Through Eqs. (1) and (2) we extract a direct measurement of $\kappa(q)$ from our experimental data, in the range $-10 \leq q \leq 10$. Curves $\kappa(q)$, calculated both for $I_{\perp}(t)$ and $I_{\parallel}(t)$ in Eq. (1), are reported in Fig. 2. We note that, while the curve referring to $I_{\parallel}(t)$ can be considered roughly linear (at least within the experimental accuracy), the curve refer-

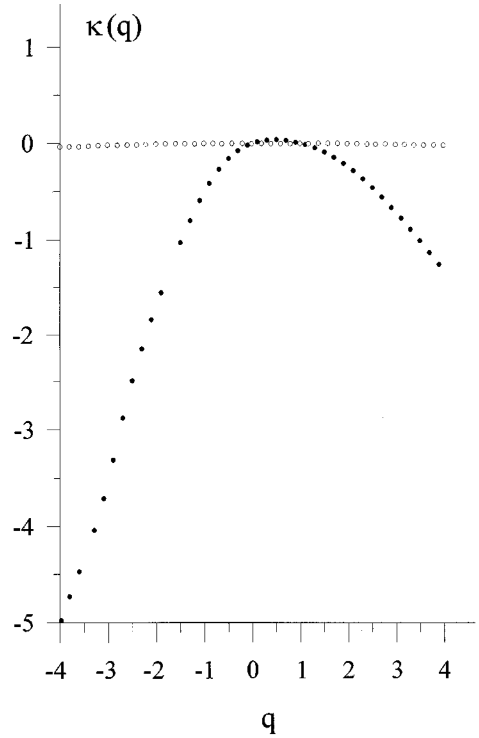

FIG. 2. Plot of the scaling exponent $\kappa(q)$ vs the qth power of $S_{\Delta t}(I,t)$. Dark and white dots refer, respectively, to $I_\perp$ and $I_\parallel$.

ring to $I_\perp(t)$ is nonlinear, thus indicating that only the attractor of $I_\perp(t)$ presents a multifractal structure. Since we have already studied the global structure of the attractors both for $I_\parallel(t)$ and $I_\perp(t)$ [9], in the following we restrict our attention to this last quantity. The multifractal structure of $I_\perp(t)$ is described by the curve $f(\alpha)$ given in Fig. 3. As is shown, the singularity spectrum assumes its maximum value $f(\alpha)\simeq 1$ when $\alpha(q=0)\simeq 1.17$, say the singularity α that lies in the majority of the box is greater than 1. There are two characteristic points of the curve $f(\alpha)$, say $f(\alpha=1)\simeq 0.97$ (which is the dimension of the set where all the singularities are located), and $f(\alpha)=\alpha=D_1\simeq 0.90$ (which represents the dimension of the set where the measure concentrates asymptotically for $\Delta t\rightarrow 0$). Both sets are almost space filling. The minimum value of α, which corresponds to $f(\alpha)=0$, is

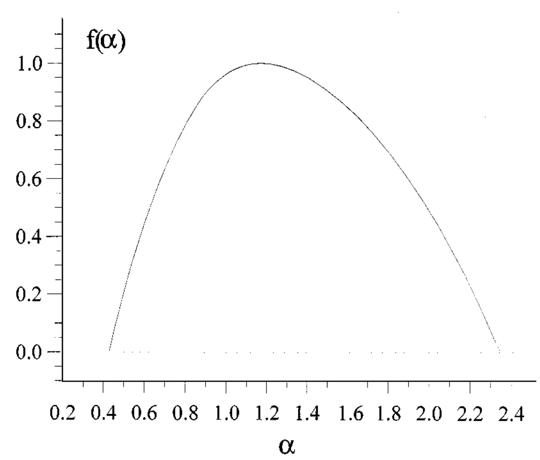

FIG. 3. The singularity spectrum $f(\alpha)$ calculated for $I_\perp$.

$\alpha_{\mathrm{min}}\simeq 0.43$, and represents the Hölder exponent corresponding to the largest singularity. Moreover the dynamics shows regions of latent singularities [11] defined by $f(\alpha)<0$ and $\alpha>0$. In our case this region occurs for $\alpha\leqslant 0.43$ and $\alpha\geqslant 2.34$. The condition of negative dimension, as shown by Mandelbrot, means that on average there is less than one box in a sample corresponding to these values of α. Finally, to characterize the intermittency, it is useful to introduce the intermittency exponent μ as the rate of increase of the tails of the probability distribution function with respect to a Gaussian function [12]. The exponent μ is closely related [5] to the scaling exponents $\kappa(q)$ through $\mu=-[d^2\kappa(q)/dq^2]_{q=0}$. In our case the intermittency exponent holds almost high $\mu\simeq 0.48\pm 0.04$, thus showing a strong intermittency. As a comparison, the intermittency exponent in the measurement of the dissipated energy in ordinary fluid flows [5] results to be $\mu\simeq 0.26\pm 0.03$.

We would stress that, to our knowledge, our data analysis represents the first experimental evidence of intermittency in the study of the NLC molecular director reorientation, which can be described by the multifractal geometry. Previous theoretical approaches to the problem at hand face only the linear regime in the phenomenon, and are then inadequate to represent the richness we have found. In this sense our analysis in terms of the probability measure on various scales could open another way to a theoretical understanding of molecular director reorientation. In fact, even if the phenomenon under examination is very complicated (as in our case), the gross features of the dynamical behavior can be achieved by simple models. Let us consider the following example which is intended only to illustrate qualitatively how the interaction between radiation and director fields can produce multifractal fluctuations of $I_\perp$. The optically induced reorientation of the molecular director can be interpreted in terms of the exchange of the angular momentum and energy between the radiation and the NLC. Let us denote by $Q(t)$ the random time series involved in the physical process. Since the phenomenon is in general nonlinear, we conjecture the existence of a kind of not homogeneous ‘‘redistribution’’ of $Q(t)$ on different scales. This could be the source of intermittency, as it happens in fluid flows for the energy flux [5]. In fact if we introduce the subsets T_i at the various scales $\tau/T=2^{-n}(n=0,1,2,\ldots)$, the probability measure $\Psi_\tau(Q)$ derived from $Q(t)$ can be described through the product of n breakage coefficients $0\leqslant\chi_i<1$,

$$\Psi_\tau(Q)=\prod_{i=1}^{n}\chi_i .$$

By supposing that the multipliers χ_i have a unique probability distribution function $P(\chi)$, and that multipliers at the nth level are statistically independent of those at the previous level, the scaling exponents $\kappa(q)$ of the various moments of the probability measure can be immediately recognized to be

$$\kappa(q)=-\ln_2\left\{\int\chi^q P(\chi)d\chi\right\}.$$

This is the simplest form we can obtain for the scaling exponents (a more sophisticated approach can be done by relaxing the two assumptions we have made). Although there are several chances for $P(\chi)$, the simplest reliable form is given in terms of the sum of two δ functions in which the breakage coefficient can assume only two values, say p and $(1-p)$. Then one obtains the usual p model

$$D_q = \ln_2[p^q + (1-p)^q]/(1-q).$$

The parameter $0<p<1$ gives a measure of the nonhomogeneity of the redistribution of the measure. When $p=\frac{1}{2}$, the process is not intermittent, and $\kappa(q)$ is linear in q. We have made a fit of our data by this model (not shown here), obtaining a good agreement in the range $-8 \leqslant q \leqslant 3$ for the best-fit value $p \simeq 0.193 \pm 0.002$, corresponding to a strong intermittent process [5]. Then in spite of our ignorance about $Q(t)$, the simple model provides a good agreement. Obviously this model, that is intended only to illustrate a suitable approach to the problem, does not address the richest dynamic process involved in the experimental situations. An attempt to look for a more satisfactory model in such a perspective is actually in progress and will be reported in a future paper.

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